1. Limit Definition of the Derivative

Using the limit definition of the derivative, find the derivative of the following functions.

(1) $f(x) = x^3$
(2) $g(x) = \frac{1}{x^2}$ (Hint: Find the common denominator, then expand)
(3) $h(x) = \frac{1}{x} + x^2 + 3$
(4) $f(x) = (5x+2)(3x-1)$

2. Derivative Rules

Using the differentiating rules (power, addition, scalar, multiplication, quotient, chain) find the derivative of the following functions:

(1) $f(x) = \frac{\sqrt{x^2-x}}{x+x^{-1}}$
(2) $h(x) = (3\csc x + 1)^{34}$
(3) $g(x) = (3x^2 + x + 1)^{10}(5x+1)^{62}$
(4) $f(x) = 4\sqrt[7]{\sin x \cdot \tan x}$

3. Implicit Differentiation

(1) Find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ for the implicit function $\frac{y}{x} = xy^4 + 1$.
(2) Find $\frac{ds}{dt}$ for the implicit function $s + \sin t = s \cdot t$.
(3) Find the equation for the line tangent to the curve $\cos(x \cdot \tan y) = y^2$ at the point $(0, 1)$.

4. Intervals of Increase and Decrease

Find the intervals of increase and decrease of the following functions:

(1) $f(x) = \frac{10x}{x^2+x+1} - 3$
(2) $g(x) = x^2 + 2x + 5$
(3) $h(x) = (2x+5)^2(3x-1)^3$

5. The Mean Value Theorem

(1) A company introduces a new product for which the number of units sold, S is given by $S(t) = 200(5 - \frac{9}{2+t})$ where t is time in months.
   (a) Find the average rate of change of units sold in the first year.
   (b) What theorem tells us that $S'(t)$ will equal the average rate of change at some point during the first year? During what month does $S'(t)$ equal its average rate of change?

6. Proofs

(1) Use derivative rules to prove the quotient rule (you may use every basic derivative rule, except for the quotient rule). That is, show that $\frac{d}{dx}(\frac{f(x)}{g(x)}) = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$.
(2) Prove the Reciprocal Identity: $\frac{d}{dx}(\frac{1}{f(x)}) = -\frac{f'(x)}{f(x)^2}$.
(3) Using derivative rules, and the derivative of $\sin x$ and $\cos x$, prove that $\frac{d}{dx}(\sec x) = \sec x \tan x$.